

Chapter Three

Solving Equations and Problem Solving

Section 3.1

Simplifying Algebraic Expressions

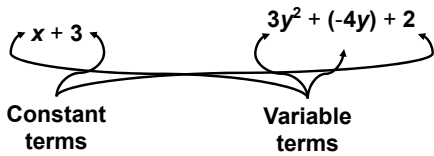
In algebra letters called variables represent numbers.

The addends of an algebraic expression are called the terms of the expression.

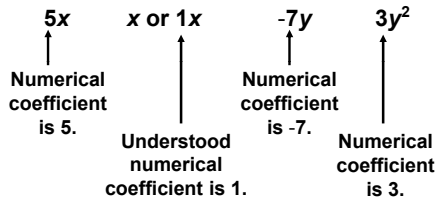
$$\begin{array}{c} x + 3 \\ \uparrow \quad \uparrow \\ \text{2 terms} \end{array}$$

$$\begin{array}{c} 3y^2 + (-4y) + 2 \\ \uparrow \quad \uparrow \quad \uparrow \\ \text{3 terms} \end{array}$$

A term that is only a number is called a constant term, or simply a constant. A term that contains a variable is called a variable term.



The number factor of a variable term is called the numerical coefficient. A numerical coefficient of 1 is usually not written.



Terms that are exactly the same, except that they may have different numerical coefficients are called like terms.

Like Terms	Unlike Terms
$3x, 2x$	$5x, x^2$
$-6y, 2y, y$	$7x, 7y$
$-3, 4$	$5y, 5$
$2ab^2, -5b^2a$	$6a, ab$

The order of the variables does not have to be the same.

A sum or difference of like terms can be simplified using the distributive property.

Distributive Property

If a , b , and c are numbers, then

$$ac + bc = (a + b)c$$

Also,

$$ac - bc = (a - b)c$$

By the distributive property,

$$7x + 5x = (7 + 5)x \\ = 12x$$

This is an example of combining like terms.

An algebraic expression is simplified when all like terms have been combined.

The commutative and associative properties of addition and multiplication help simplify expressions.

Properties of Addition and Multiplication

If a , b , and c are numbers, then

Commutative Property of Addition

$$a + b = b + a$$

Commutative Property of Multiplication

$$a \cdot b = b \cdot a$$

The order of adding or multiplying two numbers can be changed without changing their sum or product.

The grouping of numbers in addition or multiplication can be changed without changing their sum or product.

Associative Property of Addition

$$(a + b) + c = a + (b + c)$$

Associative Property of Multiplication

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Helpful Hint

Examples of Commutative and Associative Properties of Addition and Multiplication

$4 + 3 = 3 + 4$ Commutative property of Addition

$6 \cdot 9 = 9 \cdot 6$ Commutative property of Multiplication

$(3 + 5) + 2 = 3 + (5 + 2)$ Associative property of Addition

$(7 \cdot 1) \cdot 8 = 7 \cdot (1 \cdot 8)$ Associative property of Multiplication

We can also use the distributive property to multiply expressions.

The distributive property says that multiplication distributes over addition and subtraction.

$$2(5 + x) = 2 \cdot 5 + 2 \cdot x = 10 + 2x$$

or

$$2(5 - x) = 2 \cdot 5 - 2 \cdot x = 10 - 2x$$

To simply expressions, use the distributive property first to multiply and then combine any like terms.

Simplify: $3(5 + x) - 17$

$$\begin{aligned}
 3(5 + x) - 17 &= 3 \cdot 5 + 3 \cdot x + (-17) && \text{Apply the distributive property} \\
 &= 15 + 3x + (-17) && \text{Multiply} \\
 &= 3x + (-2) \text{ or } 3x - 2 && \text{Combine like terms}
 \end{aligned}$$

Helpful Hint 3 is *not* distributed to the -17 since -17 is not within the parentheses.

Finding Perimeter

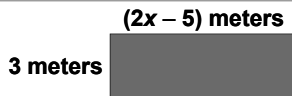


Perimeter is the distance around the figure.

$$\begin{aligned}
 \text{Perimeter} &= 3z + 7z + 9z \\
 &= 19z \text{ feet}
 \end{aligned}$$

Don't forget to insert proper units.

Finding Area



$A = \text{length} \cdot \text{width}$

$$\begin{aligned}
 &= 3(2x - 5) \\
 &= 6x - 15 \text{ square meters}
 \end{aligned}$$

Don't forget to insert proper units.

Helpful Hint

Don't forget . . .

Area:
• surface enclosed
• measured in square units

Perimeter:
• distance around
• measured in units

Section 3.2

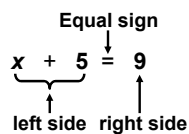
Solving Equations: The Addition and Multiplication Properties

Equation vs. Expression

Statements like $5 + 2 = 7$ are called equations.

An equation is of the form
expression = expression

An equation can be labeled as



Addition Property of Equality

Let a , b , and c represent numbers.
If $a = b$, then

$$a + c = b + c$$

and

$$a - c = b - c$$

In other words, the same number may be added to or subtracted from both sides of an equation without changing the solution of the equation.

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Multiplication Property of Equality

Let a , b , and c represent numbers and let $c \neq 0$. If $a = b$, then

$$a \cdot c = b \cdot c \quad \text{and} \quad \frac{a}{c} = \frac{b}{c}$$

In other words, both sides of an equation may be multiplied or divided by the same nonzero number without changing the solution of the equation.

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Solve for x .

$$x - 4 = 3$$

To solve the equation for x , we need to rewrite the equation in the form
 $x = \text{number}$.

To do so, we add 4 to both sides of the equation.

$$x - 4 = 3$$

$$x - 4 + 4 = 3 + 4 \quad \text{Add 4 to both sides.}$$

$$x = 7 \quad \text{Simplify.}$$

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Check

To check, replace x with 7 in the original equation.

$$x - 4 = 3 \quad \text{Original equation}$$

$$7 - 4 \stackrel{?}{=} 3 \quad \text{Replace } x \text{ with 7.}$$

$$3 = 3 \quad \text{True.}$$

Since $3 = 3$ is a true statement, 7 is the solution of the equation.

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Solve for x

$$4x = 8$$

To solve the equation for x , notice that 4 is *multiplied* by x .

To get x alone, we *divide* both sides of the equation by 4 and then simplify.

$$\frac{4x}{4} = \frac{8}{4}$$

$$1 \cdot x = 2 \quad \text{or} \quad x = 2$$

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Check

To check, replace x with 2 in the original equation.

$$4x = 8 \quad \text{Original equation}$$

$$4 \cdot 2 \stackrel{?}{=} 8 \quad \text{Let } x = 2.$$

$$8 = 8 \quad \text{True.}$$

The solution is 2.

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Using Both Properties to Solve Equations

$$2(2x - 3) = 10$$

Use the distributive property to simplify the left side.

$$4x - 6 = 10$$

Add 6 to both sides of the equation

$$4x - 6 + 6 = 10 + 6$$

$$4x = 16$$

Divide both sides by 4.

$$x = 4$$

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Check

To check, replace x with 4 in the original equation.

$$2(2x - 3) = 10 \quad \text{Original equation}$$

$$2(2 \cdot 4 - 3) \stackrel{?}{=} 10 \quad \text{Let } x = 4.$$

$$2(8 - 3) \stackrel{?}{=} 10$$

$$(2)5 = 10 \quad \text{True.}$$

The solution is 4.

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Section 3.3

Solving Linear Equations in One Variable

Linear Equations in One Variable

$3x - 2 = 7$ is called a linear equation or first degree equation in one variable.

The exponent on each x is 1 and there is no variable below a fraction bar.

It is an equation in one variable because it contains one variable, x .

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Helpful Hint

Make sure you understand which property to use to solve an equation.

Addition

$$x + 5 = 8$$

To undo addition of 5, we subtract 5 from both sides.

$$x + 5 - 5 = 8 - 5$$

Use Addition Property of Equality

$$x = 3$$

Multiplication

$$3x = 12$$

To undo multiplication of 3, we divide both sides by 3.

$$\frac{3x}{3} = \frac{12}{3}$$

Use Multiplication Property of Equality

$$x = 4$$

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Steps for Solving an Equation

- Step 1. If parentheses are present, use the distributive property.
- Step 2. Combine any like terms on each side of the equation.
- Step 3. Use the addition property to rewrite the equation so that the variable terms are on one side of the equation and constant terms are on the other side.
- Step 4. Use the multiplication property of equality to divide both sides by the numerical coefficient of x to solve.
- Step 5. Check the solution in the *original equation*.

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Key Words or Phrases that translate to an equal sign when writing sentences as equations.

Key Words or Phrases	Sentences	Equations
equals	5 equals 2 plus 3.	$5 = 2 + 3$
gives	The quotient of 8 and 4 gives 2.	$\frac{8}{4} = 2$
is/was/will be	x is 5.	$x = 5$
yields	y plus 6 yields 15.	$y + 6 = 15$
amounts to	Twice x amounts to - 8.	$2x = - 8$
is equal to	36 is equal to 4 times 9.	$36 = 4(9)$

Section 3.4

Linear Equations in One Variable and Problem Solving

Problem-Solving Steps

1. UNDERSTAND the problem. During this step, become comfortable with the problem. Some ways of doing this are:

- **Read and reread the problem.**
- **Choose a variable to represent the unknown.**
- **Construct a drawing.**
- **Propose a solution and check it. Pay careful attention to how you check your proposed solution. This will help when writing an equation to model the problem.**

Problem-Solving Steps . . .

2. **TRANSLATE** the problem into an equation.
3. **SOLVE** the equation.
4. **INTERPRET** the results. *Check* the proposed solution in the stated problem and *state* your conclusion.
