# Learning Objectives for Section 1.1 Linear Equations and Inequalities

After this lecture and the assigned homework, you should be able to

- solve linear equations.
- solve linear inequalities.
- use interval notation correctly.
- solve applications involving linear equations and inequalities.

# Linear Equations, Standard Form

In general, a **first-degree, or linear, equation** in one variable is any equation that can be written in the form

$$ax + b = 0, \qquad a \neq 0$$

This is called the **standard form** of the linear equation.

For example, the equation

$$3 - 2(x + 3) = \frac{x}{3} - 5$$

is a **linear** equation because it can be converted to standard form by clearing of fractions and simplifying.

# Writing a Linear Equation into Standard Form

**Example:** Write the linear equation in standard form:

$$3 - 2(x + 3) = \frac{x}{3} - 5$$

# Equality Properties

An equivalent equation will result if

1. The same quantity is added to or subtracted from each side of a given equation.

2. Each side of a given equation is multiplied by or divided by the same nonzero quantity.

To **solve** a linear equation, we perform these operations on the equation to obtain simpler equivalent forms, until we obtain an equation with an obvious solution.

# Example of Solving a Linear Equation

**Example 1:** Solve and check.

$$\frac{x-3}{2}-\frac{x}{5}=3$$



*The check is on the next page…*

# Example of Solving a Linear Equation

Checking: $$\frac{x-3}{2}-\frac{x}{5}=3$$

# Solving a Formula for a Particular Variable

**Example 2:** Solve $A = P + Prt$, *solve for r*

# Linear Inequalities

If the equality symbol = in a linear equation is replaced by an inequality symbol ($<$, $>$, $\leq$, or $\geq$), the resulting expression is called a **first-degree, or linear, inequality**. For example

$$5 \leq \; 1 - 3x \;\; 2 + \frac{x}{2}$$

is a linear inequality.

# Example for Solving a Linear equality

**Example 3:** Solve the inequality, graph the solution and write the solution in interval notation.

**Solve,** $5 + 4x - 7 = 4x - 2 - x$

# Example for Solving a Linear equality

**Example 4:** Solve the inequality, graph the solution and write the solution in interval notation.

**Solve, $11 + 3x - 7 = 6x + 5 - 3x$**

# Inequality Properties

- The direction of an inequality **will remain the same** if
  1. Any real number is added to or subtracted from both sides.
  2. Both sides are multiplied or divided by a positive number.
- The direction of an inequality **will reverse** if
  - Both sides are multiplied or divided by a negative number.

*Note: Multiplication by 0 and division by 0 are NOT allowed.*

# Example for Solving a Linear Inequality

**Example:** Solve the inequality and graph the solution.

$$2(2x + 3) < 6(x - 2) + 10$$

# Double Inequalities

If $a < b$, the **double inequality** $a < x < b$ means that $a < x$ and $x < b$. That is, $x$ is between $a$ and $b$.

# Interval and Inequality Notation

**Interval notation** is also used to describe sets defined by single or double inequalities, as shown in the following table.

| Inequality | Interval | Graph |
|---|---|---|
| $a \le x \le b$ | [a, b] | |
| $a \le x < b$ | [a,b) | |
| $a < x \le b$ | (a,b] | |
| $a < x < b$ | (a,b) | |
| $x \le a$ | (-∞, a] | |
| $x < a$ | (-∞, a) | |
| $x \ge b$ | [b, ∞) | |
| $x > b$ | (b, ∞) | |

# Interval and Inequality Notation and Line Graphs

**Example 1:** Write [-5, 2) as a double inequality and graph.

$-5 \leq x < 2$

**Example 2:** Write $x \geq -2$ in interval notation and graph.

$[-2, \infty)$

# Example for Solving a Linear Inequality

**Example 4:** Solve the inequality, graph the solution.

**Solve: (x-3)/2 < -5**

First, let us clear out the "/2" by multiplying both sides by 2:

(x-3)/2 **×2** < -5 **×2**

(x-3) < -10

Now add 3 to both sides:

x-3 **+ 3** < -10 + **3**

x < -7

And that is our solution: **x < -7**

# Example for Solving a Double Linear Inequality

**Example 5:** Solve the double inequality, graph the solution and write the solution in interval notation.

$$-9 \leq 3x < 12$$

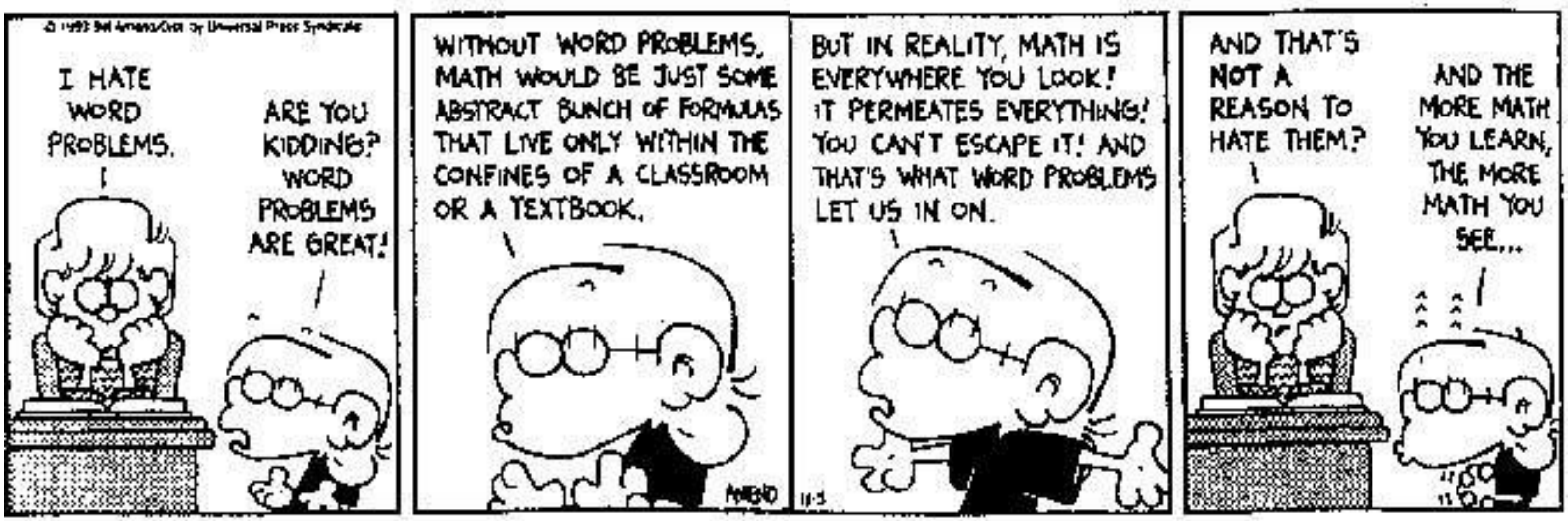
# Example for Solving a Double Linear Inequality

**Example 6:** Solve the double inequality, graph the solution and write the solution in interval notation.

$$-3 < 2x + 3 \leq 9$$

# Procedure for Solving Word Problems

1. Read the problem carefully and introduce a variable to represent an unknown quantity in the problem.
2. Identify other quantities in the problem (known or unknown) and express unknown quantities in terms of the variable you introduced in the first step.
3. Write a verbal statement using the conditions stated in the problem and then write an equivalent mathematical statement (equation or inequality.)
4. Solve the equation or inequality and answer the questions posed in the problem.
5. Check that the solution solves the original problem.



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I HATE WORD PROBLEMS.
ARE YOU KIDDING? WORD PROBLEMS ARE GREAT!
WITHOUT WORD PROBLEMS, MATH WOULD BE JUST SOME ABSTRACT BUNCH OF FORMULAS THAT LIVE ONLY WITHIN THE CONFINES OF A CLASSROOM OR A TEXTBOOK.
BUT IN REALITY, MATH IS EVERYWHERE YOU LOOK! IT PERMEATES EVERYTHING! YOU CAN'T ESCAPE IT! AND THAT'S WHAT WORD PROBLEMS LET US IN ON.
AND THAT'S NOT A REASON TO HATE THEM?
AND THE MORE MATH YOU LEARN, THE MORE MATH YOU SEE...

# Some Business Terms

**Revenue (R)-** Money taken in on the sales of an item

**Costs (C)-** The cost to produce an item.

The cost includes both fixed and variable costs.

$$C = \text{fixed costs} + \text{variable costs}$$

**Fixed costs-** expenses for rent, plant overhead, product design, setup, and promotion.

**Variable costs-** expenses dependent on the # of items produced.

# Some Business Terms

**Break-Even:** Revenue = Cost

$$R = C$$

**Profit:** $R > C$

**Loss:** $R < C$

**Revenue =** money in

**Break-Even Point = Fixed Costs/(Unit Selling Price - Variable Costs)**

# Example: Break-Even Analysis

A recording company produces compact disk (CDs). One-time fixed costs for a particular CD are $24,000; this includes costs such as recording, album design, and promotion. Variable costs amount to $6.20 per CD and include the manufacturing, distribution, and royalty costs for each disk actually manufactured and sold to a retailer. The CD is sold to retail outlets at $8.70 each. How many CDs must be manufactured and sold for the company to break even?

# Break-Even Analysis Solution (continued)

**Solution**

C(x) = Fixed cost + variable cost

C(x) = 24,000 + 6.20x

R(x) = 8.70x, where C(x) = R(x)

24,000 + 6.20x = 8.70x

24,000 = 8.70x - 6.20x => $24,000/2.5 = x

X = 9,600

# Break-Even Analysis Solution (continued)

**Check it:**

# Break-even Analysis

Assume that the financial statements for Lillian's Bakery reveal that the bakery's fixed costs are $49,000, and its variable costs per unit of production (loaf of raisin coffee cake) are $.30.

Further assume that its sales revenue is $1.00 per loaf. From this information, it can be determined that, after the $.30 per loaf variable costs are covered, each loaf sold can contribute $.70 toward covering fixed costs.

# Application Problem (Table Example)

*From text: page 12 #54*

An all-day parking meter takes only dimes and quarters. If it contains 100 coins with a total value of $14.50, how many of each type of coin are in the meter?

# Application Problem Continued…

Let x = number of quarters in the meter. Then 100-x= number of dimes in the meter.

Now, $0.25x + 0.10(100-x) = 14.50$ or
$0.25x + 10 - 0.10x = 14.50$

$0.15x=4.50$
$X=4.50/0.15=30$
Thus, there will be 30 quarters and 70 dimes