

Semester Review

Spring 2010

Linear Equations

How can you tell a linear equation from other equations?

1. A linear equation can be written in the forms:

GENERAL FORM: $Ax + By = C$

where A, B, and C are **constants** and x and y are variables.

 (numbers)

SLOPE-INTERCEPT FORM: $y = mx + b$

where m is the slope and b is the y-intercept.

POINT-SLOPE FORM: $y - y_1 = m(x - x_1)$

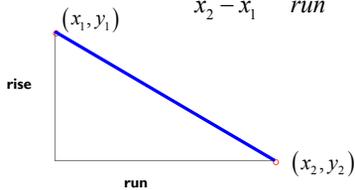
where m is the slope and (x_1, y_1) is a point.

2. A linear equation graphs a **straight line**.

▶ 2

Slope of a Line

Slope of a line: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$



Note: The slope of a line is the SAME everywhere on the line!!! You may use *any* two points on the line to find the slope.

▶

3

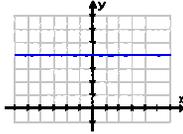
Calculating Slope



Example: Calculate the slope between the given points.

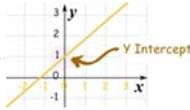
(3, 7) and (1, 7)?

$$\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 7}{1 - 3} = \frac{0}{-2} = 0$$



▶ 4

Intercepts of a Line



x-intercept: where the graph crosses the x-axis.

The coordinates are $(a, 0)$.

y-intercept: where the graph crosses the y-axis.

The coordinates are $(0, b)$.

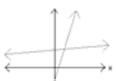
▶ 5

Recall that there are three possibilities for the manner in which the graphs of two linear equations could meet. The lines could...

intersect once not intersect at all

intersect an infinite number of times (the two lines are the same)

(we have seen this) (be parallel)



consistent
independent



inconsistent
independent



consistent
dependent

- consistent system – a system of equation with a solution
- inconsistent system – a system of equation without a solution
- dependent equations – equations that are equivalent
- independent equations – equations that are **not** equivalent

▶

Example: Use the elimination method to solve the system of linear equations.

$$\begin{cases} 4x + 6y = 5 \\ 8x - 3 = 9y \end{cases}$$

Is the system consistent or inconsistent?

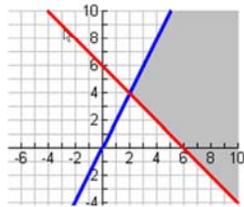
Are the equations dependent or independent?

Solve With Calc. and Check

Is it Bound/unbound?; Consistent/inconsistent and explain?
Dependent/independent and explain?; What is the solution?

Systems of Linear Inequalities that is unbound.

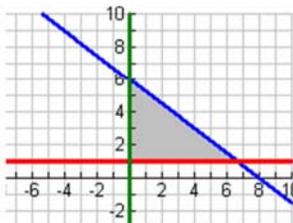
$$\begin{cases} y \leq 2x \\ y \geq -x + 6 \end{cases}$$



Systems of Linear Inequalities

Systems of Linear Inequalities that is bound.

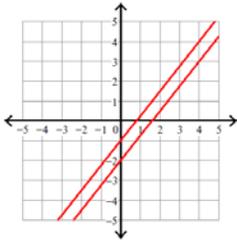
$$\begin{cases} y \leq -\frac{3}{4}x + 6 \\ y \geq 1 \\ x \geq 0 \end{cases}$$



Solve by Graphing and Check

$$3) y = \frac{5}{4}x - 2$$

$$y = \frac{5}{4}x - 1$$



No solution

Matrices

Is it Bound/unbound?; Consistent/inconsistent and explain?
Dependent/independent and explain?; What is the solution?

$$9) \begin{cases} 36x - 6y + 6z = -1 \\ 12x + 6z = -3 \\ 18x - 6y + 6z = -1 \end{cases}$$

$$4) \begin{cases} 4x - 4y + 2z = -14 \\ 4x + 2y = 14 \\ -3y + z = -10 \end{cases}$$

$$11) \begin{cases} -3z = 6 \\ 2x + y - 2z = 6 \\ -6x - 3y = -6 \end{cases}$$

Write the System of Linear Equations

$$\left[\begin{array}{ccc|c} 36 & -6 & 6 & -1 \\ 12 & 0 & 6 & -3 \\ 18 & -6 & 6 & -1 \end{array} \right]$$

Forms for a Linear System in Two Variables

Form 1: Unique Solution
(Consistent and Independent)

Standard form

$$\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array} \right]$$

Form 2: Infinitely Many Solutions
(Consistent and Dependent)

$$\left[\begin{array}{cc|c} 1 & a & b \\ 0 & 0 & 0 \end{array} \right]$$

Form 3: No Solution
(Inconsistent and Independent)

$$\left[\begin{array}{cc|c} 1 & a & b \\ 0 & 0 & c \end{array} \right]$$

▶ 13

Explain reasoning

d) Is the system consistent or inconsistent? Explain your reasoning.

The system is consistent because it has at least one solution (in this case, there are infinitely many solutions.) The ref matrix is NOT in standard form (the 2x2 matrix containing the coefficients of the variables x & y does not have 1's on the diagonal, 0's in remaining entries); thus, there is not a unique solution. Furthermore, the second equation of the ref matrix is $0x + 0y = 0 \Rightarrow 0 = 0$, which is a TRUE statement. Thus the system has infinitely many solutions.

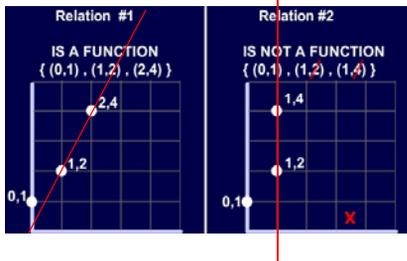
You can also tell by looking at the slopes and y-intercepts of the lines. The slopes and the y-intercepts of the lines are the same. When graphed, the two lines coincide (share all points).

e) Are the equations dependent or independent? Explain your reasoning.

Because there are infinitely many solutions to the system, the equations must be dependent. When graphed, the lines coincide (graph appears as a single line).

▶

Vertical line test for a function



On the left are the graphs of the two relations examined above. Relation #2 does not pass the vertical line test.

▶

Example (find the domain)

Find the domain of function f defined by

$$y = \frac{x^2 + x - 2}{x^2 - x - 2}$$

Solution to Example 2

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1$$

Know how to factor!!!

Example 3 (find the domain)

$$y = -\sqrt{-2x + 3}$$

The domain is all values that x can take on. I cannot have a negative inside the square root. So I'll set the insides greater-than-or-equal-to zero, and solve. The result will be my domain:

$$-2x + 3 \geq 0$$

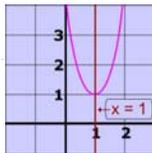
$$-2x \geq -3$$

$$2x \leq 3$$

$$x \leq 3/2$$

Then the domain is "all $x \leq 3/2$ ".

Definitions

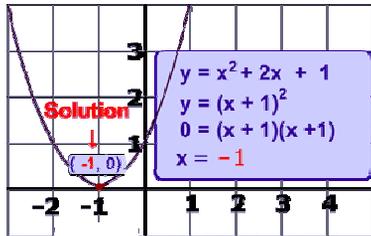


▶ Every parabola has an **axis of symmetry** which is the line that runs down its center. This line divides the graph into two perfect halves.

▶ Axis of symmetry formula is $x = -b/2a$

Solving a Quadratic Equation $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 1 Quadratic Equation: $y = x^2 + 2x + 1$ where $a = 1$, $b = 2$, and $c = 1$



Example: Break-Even Analysis

A recording company produces compact disk (CDs). One-time fixed costs for a particular CD are \$24,000; this includes costs such as recording, album design, and promotion. Variable costs amount to \$6.20 per CD and include the manufacturing, distribution, and royalty costs for each disk actually manufactured and sold to a retailer. The CD is sold to retail outlets at \$8.70 each. How many CDs must be manufactured and sold for the company to break even?

Break-Even Analysis **Solution**

(continued)

Solution

$$TC = \text{Fixed cost} + \text{variable cost}$$

$$TC = 24,000 + 6.20x$$

$$R(x) = 8.70x, \text{ where } TC = R(x)$$

$$24,000 + 6.20x = 8.70x$$

$$24,000 = 8.70x - 6.20x \Rightarrow \$24,000/2.5 = x$$

$$x = 9,600$$

Compounding Periods

- ▶ Annually: once a period
- ▶ Quarterly: Four times a period
- ▶ Semiannually: Twice a period
- ▶ Monthly: Twelve times a period

Compound Interest

- ▶ This is the compound interest formula:

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

Example of Compound Interest $A = P \left(1 + \frac{r}{n} \right)^{nt}$

An amount of \$1500.00 is deposited in a bank paying an annual interest rate of 4.3%, compounded quarterly. Find the balance after 6 years.

A. Using the formula above, with $P = 1500$, $r = 4.3/100 = 0.043$, $n = 4$, and $t = 6$:

$$A = 1500 \left(1 + \frac{0.043}{4} \right)^{4 \times 6} = 1938.84$$

So, the balance after 6 years is approximately \$1,938.84.

Example

- ▶ A financial planner recommends a plan for a client who has \$65,000 to invest. Part of the money will be invested in a mutual fund at 12% and the remainder will be invested in municipal bonds at 6.2%. The total interest earned at the end of one year will be \$6,477.60. How much will be invested at each rate?

Example 2

- ▶ A teacher wants to supplement his retirement income with investment interest. If he invests \$15,000 at 6% interest, how much more would he have to invest at 7% to achieve a goal of \$1,250 per year in supplemental income?

Some Types of Regression

Linear Regression (straight line form)- menu option 4:LinReg(ax+b)



Quadratic Regression (parabolic form)- menu option 5:QuadReg



Cubic Regression (cubic form)- menu option 6:CubicReg



Example 1: Energy consumption

The table lists US fossil fuel consumption as a percentage of total energy consumption for selected years. A linear regression model of this data is $y = -.22x + 94$ where x represents years since 1960 and y represents the corresponding percentage of fossil fuel consumption.)

| Year | 1960 | 1970 | 1980 | 1990 | 2000 |
|-----------|------|------|------|------|------|
| Prod. (%) | 93 | 94 | 89 | 86 | 86 |

Let $x = 0$ correspond to 1960.

- a) Using a graphing calculator, draw the scatterplot. Interpret the plot:
1. Form
 2. Strength
 3. Direction

28

Supply and Demand and Cost Function

- ▶ 1) Supply and Demand. Let the supply and demand functions for sugar be given by

$$p = S(q) = 1.4q - .6 \quad \text{and}$$
$$p = D(q) = -2q + 3.2,$$

where p is the price per pound and q in the quantity in thousands of pounds.

- a. Graph these on the same axes.
- b. Find the equilibrium quantity and the equilibrium price.

Functions (review p55)

- ▶ Cost function: $C = (FC) + (VC)$
- ▶ Revenue function:
 $R = (\# \text{ of items sold}) \times (\text{price per item})$
- ▶ Profit function: $P = R - C$
- ▶ Know how to write these

Examples: Solve the equation and check your result.

$$\frac{x+2}{13} = 3$$

we will clear this equation of fractions

check

Example: Solve the inequality. Graph the solution set and express your answer using interval notation.

$$1 \leq -7x + 8 \leq 15$$

graph



interval notation

Things to remember

1. Show work and check
2. Interval notation (x, y)
3. Use ruler, bring extra batteries
4. Pencils

Preparing for the test

- ▶ Review previous homework
- ▶ Review class notes
- ▶ Review concepts and definitions
- ▶ Complete the chapter review
- ▶ Place yourself in test-like conditions
- ▶ Get a good night's sleep
- ▶ Allow plenty of time to arrive for test
- ▶ Study a week out
- ▶ If you don't know something get the answer
- ▶ Show work and check
- ▶ Interval notation or ordered pair (x, y)
- ▶ Use ruler, bring extra batteries
- ▶ Pencils

Taking the test

- ▶ Read directions carefully
- ▶ Read each problem carefully
- ▶ Watch your time
- ▶ If you have time, check work and answers
- ▶ Do not turn your test in early
- ▶ If possible, double-check your work
