Probability

Chapter 5

5.1 Probability Rules

- Probability is a measure of the chance that a random behavior will occur.
- The Law of Large Numbers (pg. 224) states that as repetitions of probability experiments increases, the proportion of a certain outcome get closer to the probability outcome.
  - Flipping a coin 10 times vs 1000 times

Simple Probability

- A simple event is any single outcome from a probability experiment (roll die once). Denoted by e_i.
- The sample space of a probability experiment is the collection of all possible simple events (S = {1, 2, 3, 4, 5, 6}
- An event is any collection of outcomes from a probability experiment. An event may have one or more simple events. Events are denoted by capital letters such as E.

Properties of Probabilities

- P(E) must be between 0 and 1.
  $0 \le P(E) \le 1$
- A probability of 0 means the event is impossible.
- A probability of 1 means the event is a certainty.
- The sum of all probabilities of an event happening is 1.

Classical Examples Using A Pair of Dice (pg. 228)

- Find the probability that you roll a sum of 2. P(sum = 2)
- Find the probability that one of the dice is a five. P(5 is rolled)
- Find the probability that the sum of the dice is 7. P(sum = 7)
- Find the probability that the sum of the dice is greater than 6. P(sum >6)

Empirical Probabilities

- Probabilites found by the empirical approach require a probability experiment to be conducted.
- P(E) $\approx$ relative frequency of E =

$$\frac{\text{frequency of E}}{\text{number of trials of experiment}}$$

Problems to Work

- Pg. 233 - #11
- Pg. 234 - #27- 30, 33
- Pg. 235 - #39
- Pg. 236 - #47

5.2 The Addition Rule; Complements

- E *and* F means event the consists of simple events that belong to both E and F.
- E *or* F means the event consists of simple events that belong to either E or F or both.
- P(E or F) = P(E) + P(F) – P(E and F)

Mutually Exclusive or (Disjointed) Events

- Events which preclude the other event from happening.
- P(E and F) = 0

Complements

- P(E)' = 1 – P(E)
- Use the complement, whenever it is easier

Problems to Work

- Pg. 245 - #5, 6, 8, 10, 12
- Pg. 246 - #14, -20 evens
- Pg. 246 - #21-24, 29
- Pg. 247 - #32
- Pg. 248 - #41

5. 3 Independence and The Multiplication Rule

- Two events E and F are independent if the occurrence of event E in a probability experiment does not affect the probability of event F. They are dependent if one affects the probability of the other.
- Multiplication Rule for independent events

  P(E and F) = P(E) ▪ P(F)

Computing Probabilities of Independent Events

- Example 3 on pg 252
- Example 4 on pg. 253
- Problems to Work
  - Pg. 254 - #7, 17

5.4 Conditional Probability

- Conditional Probability P(F|E) is read the "probability of event F given event E. It is the probability that event F occurs, given that event E has occurred.
- Go to Example 1 on pg. 257
- Conditional probabilities reduce the size of the sample space under consideration.
- Go to Example 2 on pg. 258

Conditional Probability Rule

- Conditional Probability Rule

$$P(F|E) = \frac{P(E \text{ and } F)}{P(E)} = \frac{N(E \text{ and } F)}{N(E)}$$

- Pg. 263 - # 13, 18
- General Multiplication Rule

$$P(E \text{ and } F) = P(E) \cdot P(F|E)$$

- Go to example 6 on pg. 261
- Pg. 263 - # 18

Conditional Probability and Independence

- If a small random samples are taken from a large population it is reasonable to assume independence of the events.
- Two events E and F are independent if P(E|F) = P(E) or, equivalently if P(F|E)= P(F).
- On pg 265 - #42

5.5 Counting Techniques

- Multiplication Rule of Counting (aka Sequential Counting Principle)

  (See pg. 268)
- Examples 2 and 3 on pg. 268
- Do Problems pg. 277 - # 32 , 40